Perturbative Pions and the Effective Range Expansion

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In this talk the Q counting scheme to implement effective field theory is discussed. It is pointed out that there are two small mass scales in the problem m_π and $1/a$ with $1/a \ll m_\pi$. It is argued that while the expansion based on $1/a$ being small compared to the underlying short distance scales works well, the chiral expansion may not. The coefficients of the effective range expansion are sensitive to the chiral physics and are very poorly described in Q counting at lowest nontrivial order. A "shape function" is introduced which again is sensitive to pionic physics and insensitive to fitting procedures. It is also poorly described in Q counting.

1 Introduction

The Q counting scheme introduced just over one year ago by Kaplan, Savage and Wise (KSW) [1] represents an important advance in the development of effective field theory techniques for nuclear physics. The approach is systematic, it builds in approximate chiral symmetry and chiral power counting, it solves Weinberg's [2] "unnatural scattering length problem", and, in principle, it provides *a priori* estimates of errors for observables since one works to fixed order in Q/Λ. This last feature is extremely important since in Weinberg's initial formulation [2] it was unclear how to make such error estimates. [a] To a considerable practical extent the major advantage to using EFT technology as opposed to unsystematic models is the ability to specify the accuracy of one's predictions. Thus, in this talk, I will focus entirely on Q counting and not on the many beautifully accurate calculations based on Weinberg's approach implemented with a finite cutoff [3].

The scheme introduced by KSW may be divided up into two parts. The first is Q counting and the second is a set of technical tricks to implement the Q counting. These technical tricks are rather unusual; they are based on

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[a] Of course, the principal difference between Q counting and the Weinberg scheme is that in Weinberg's approach the potential is iterated to all orders whereas in Q counting only the leading term is iterated to all orders; all other operators are treated perturbatively. If Q counting is valid, a scheme like Weinberg's if it can be systemat

ically implemented will, at worst, add uncontrolled higher order contributions which do not spoil the systematic error estimates. Thus even if one is using Weinberg's approach, if one wants to make simple error estimates one may resort to Q counting for the error estimate, provided the system is in a regime where it is valid. This view of error estimation in the Weinberg scheme is featured prominently in the discussions at this workshop.

the so-called PDS scheme for doing subtractions in dimensional regularization. Apart from the very peculiar prescriptions required (subtracting the poles as $d = 3$!) the formalism is not completely transparent in terms of the physics. Of course, provided the scheme is consistent and we are in the regime for which Q counting is valid, one should get the same results for any scheme which implements the Q counting. Two other approaches to Q counting have been tried—the OS scheme with dimensional regularization of Mehan and Stewart[4] and a cutoff scheme in configuration space [5]. All the schemes give the same results at fixed order in Q counting. Thus, ultimately the physics turns on whether or not the Q counting is working.

The Q counting scheme is straightforward:

$$Q \sim 1/a$$
$$Q \sim m_\pi$$
$$Q \sim p \tag{1}$$

where a is the scattering length–either singlet or triplet, and p represents external momenta. For partisans of the OS scheme, you can just as well use $\gamma = \sqrt{MB}$ where B is the magnitude of the binding energy, in place of $1/a$. All other scales are assumed to be heavy and will collectively be denoted as Λ. The expansion implied by Q counting is in Q/Λ. Now the key point of Q counting is that since p, m_π and $1/a$ are all of the same order, at any order in Q counting we have $1/(pa)$ and p/m_π to all orders. One important feature of Q counting is that while one needs to iterate the lowest order contact term to all orders to get a consistent result, all higher order contributions, including those from the pion can be treated perturbatively.

Q counting has been used to calculate a number of observables [1,4,6] and generally seems to have real predictive power. At first sight this would appear to rule out the possibility that $m_\pi \sim \Lambda$. However, most "vanilla" observables principally test the $1/(a\Lambda)$ part of the theory. Clearly, it is important to identify observables which are principally sensitive to the m_π/Λ parts of the theory and to rigorously test the chiral expansion. Recall that only the chiral part of the Q counting is really understood in terms of QCD. In Q counting, the small value of $1/a$ is treated as an essential fact of life that we cannot ignore. ¿From the QCD level, however, this fact of life is seen as essentially an accident. In contrast, the chiral physics is understood directly in terms of the small quark masses in the QCD Lagrangian along with spontaneous symmetry breaking. The central theme of this talk is that the effective range expansion—namely the expansion of $p \cot(\delta)$ as a power series in energy—is a good place to test whether the chiral part of Q counting is under control.

The effective range expansion (ERE) is a good place to look at pionic effects for a number of reasons. The expansion may be written:

$$p\cot(\delta) \;=\; -\frac{1}{a} \;+\; \frac{1}{2}r_e\,p^2 \;+\; v_2 p^4 \;+\; v_3 p^6 \;+\; v_4 p^8 \;+\; \ldots \tag{2}$$

Simple Q counting shows that the scattering length term is order Q^1 while all other terms in the ERE are at least order Q^2. When one includes pions explicitly the same counting holds; all terms except the first are $\mathcal{O}(Q^2)$. It is important to note the distinction between the Q expansion and the momentum expansion in the ERE. They differ precisely because the Q expansion has k/m_π and $1/(ka)$ to all orders while in the momentum expansion they are multiplied out. One immediately deduces that $v_n \sim Q^{-2n+2}$. Moreover all of the v_i coefficients in the expansion diverge in the chiral limit of $m_\pi \to 0$. Hence they are pion dominated quantities and should provide a test of the chiral part of Q counting.

2 Scales in Nuclear Physics

Before coming to the effective range expansion in this approach, it is useful to look a bit more closely at the various scales underlying Q counting. Formally, there are two light scales intrinsic to the problem, $1/a$ and m_π. In Q counting they are both formally of the same order namely $\mathcal{O}(Q)$. But emprically for both the triplet and singlet channel,

$$m_\pi \gg 1/a \tag{3}$$

with $m_\pi a \approx 4$ for the triplet channel and $m_\pi a \approx 16$ for the singlet channel. This raises the following logical possibility;

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\begin{aligned}
m_\pi &\sim \Lambda \\
1/a &\ll \Lambda
\end{aligned}
\tag{4}
$$

i.e., that there is no scale separation between pionic scales and the "short distance scales" but there is a good scale separation between them and $1/a$.

Of course, an immediate prejudice is that the first relation in eqs. (4) must be wrong; chiral scales are intrinsically long compared to typical hadronic scales. However, what is relevant here for Λ is not hadronic scales, but *nuclear* scales. The relationship of nuclear scales to QCD is quite obscure, but it is certainly true that typical nuclear mass scales are much lower than typical hadronic scales. If conditions in eqs. (4) turn out be true one would expect that the parts of the theory which depend on $1/(a\Lambda)$ will work quite well, while the parts which depend on m_π/Λ will converge slowly or not at all.

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Note, if it turns out that $m_\pi \sim \Lambda$, there is nothing in principle wrong with the Q counting formalism and PDS. It would simply not be useful for real world situations. Of course, one could play God and consider a world in which the pion is much lighter than in nature and then one would have real predictive power. In principle, if lattice technology improves, one could calculate properties in such an artificial world from first principles and could use the Q counting technology to make predictions for this world.

Before looking at explicit calculations, we should ask whether it is reasonable to suppose that $m_\pi \sim \Lambda$. Ultimately, this question comes down to whether $1/a$ and m_π are the only light scales in nuclear physics. The answer appears to be "no". Numerically:

$$\frac{1}{m_\pi} \approx 1.5\text{fm} \ \ a^s \approx -23\text{fm} \ \ a^t \approx 6\text{fm}$$

where the superscript s (t) refers to the singlet (triplet) channel. Compare these with the effective ranges:

$$r_e^s \approx 2.7\text{fm} \ \ r_e^t \approx 1.6\text{fm}$$

It is apparent that $m_\pi r_e \sim 1$; if $r_e \sim 1/\Lambda$ there is a serious potential problem.

Of course, it is possible that the large numerical size of r_e is itself a reflection of chiral physics. For example, if $r_e \sim 1/m_\pi$ there would be no problem. One can use Q counting itself to answer the question of how r_e behaves. At leading nonvanishing order it is given by[4,5]

$$r_e = \mathcal{O}(\Lambda) + \frac{g_A^2 M}{4\pi f_\pi^2 a^2 m_\pi^2} - \frac{g_A^2 M}{3\pi f_\pi^2 a m_\pi} = \mathcal{O}(Q^0) \tag{5}$$

Note that although there *is* a chiral enhancement—the last two terms diverge in the chiral limit—it is compensated for by factors of the scattering length in the denominator. Thus, one expects in the context of Q counting the effective range to be a short distance scale. In practice, however, it is larger than $1/m_\pi$. This suggests, but does not prove, that the chiral scale is not well separated from "short distance" scales.

There is another way to see that "short distance" scales may be comparable to the pion mass scale. Consider the typical scales in so-called realistic N-N potential models, *i.e., those which fit the scattering data.* If you look at the non–one-pion-exchange part of the potential it is, in fact, larger than the OPEP potential for a distance less than $\sim 1.5-2$ fm. Since $1/m_\pi$ is comparable to, or shorter than, this distance we again appear to have evidence that non-chiral supposedly short distance scales are comparable to $1/M_\pi$. One might

argue that the central potential contains two-pion-exchange physics (suitably reparameterized) in the potential model. However, if Q counting is valid, that contribution is chirally suppressed.

This argument appears to be model dependent as it is based on "typical" potential models. There is a model independent way to constrain the short distance physics.[7] Consider *any* nonrelativistic potential, including possible non-local potentials. Write the potential as the sum of an OPEP potential and some short distance potential with the constraint that the short distance potential vanishes beyond some distance R:

$$V(\vec{r}, \vec{r'}) \,=\, V_{\text{OPEP}}(\vec{r}) \, \delta(\vec{r} - \vec{r'}) + V_{\text{short}}(\vec{r}, \vec{r'})$$

$$\text{with} \quad V_{\text{short}}(\vec{r}, \vec{r'}) = 0 \quad \text{for} \quad r, r' > R. \tag{6}$$

Now suppose that this potential is inserted to a Schrödinger equation and used to solve for singlet phase shifts. A remarkable theorem can then be proved, namely that if the short distance potential fits the scattering length and effective range there is a minimum value for R. For real world values one can deduce the $R > 1.1$ fm [7]. Moreover the derivation of this bound shows that it is unsaturatable so one expects R to be significantly more than than 1.1 fm. From this one deduces that substantial contributions to the scattering come from "short distance" contributions which come from separations of greater than 1.1 fm. Recalling that $1/m_\pi \approx 1.4$ fm, we see immediately that there is no significant scale separation between m_π and the scales fixing the overall range of the nonpionic part of the potential.

While the preceeding arguments do not decisively prove that $\Lambda \sim m_\pi$ they certainly show that it is not implausible.

3 Q Counting and Cutoffs

Now the problem comes down to computing $p \cot(\delta)$. This can be done in PDS as in ref. 1. For the present purpose it is instructive to consider the cutoff calculation and we take our discussion from ref. 5. The essential physical idea in this approach is to implement the separation of long distance physics from short distance physics directly in configuration space. A radius, R, is introduced as a matching point between long and short distance effects; renormalization group invariance requires that physical quantities must be independent of R. It is important, however, that R be chosen large enough so that essentially all of the effects of the short distance physics is contained within R. The potential is divided into the sum of two pieces, a short distance part which vanishes for $r > R$ and a long distance part which vanishes for $r < R$. At

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R, the information about short distance effects is entirely contained in the energy dependence of the logarithmic derivative (with respect to position) of the wave function at R. Thus, provided we can parameterize this information systematically, we can formulate the problem in a way which is insensitive to the details of the short distance part of the potential. This insensitivity to the details of the short distance physics is at the core of why effective field theory works. For $r > R$, the Schrödinger equation is solved subject to the boundary conditions at R. For s wave scattering, the wave function at R may be parameterized as $A\sin(kr + \delta_0)$; the energy dependence of the logarithmic derivative is independent of A and can be expressed in terms of an expansion similar to an effective range expansion:

$$p\cot(\delta_0) = -1/a_{\text{short}} + 1/2\,r_e^0\,p^2 + v_2^0\,p^4 + v_3^0\,p^6 + v_4^0\,p^8 + \ldots \qquad (7)$$

Power counting in Q for s wave scattering can be implemented straightforwardly. All of the coefficients in the preceding expansion are assumed to be order Q^0 except the first term $(-1/a_{\text{short}})$ which will be taken to be order Q^1 to reflect the unnaturally large scale of the scattering length. Power counting for the long range part of the potential simply follows Weinberg's analysis[2], with the proviso that the potentials are only used for $r > R$. At order Q^2 in $p\cot(\delta)$, only the simple one-pion-exchange contribution to the V_{long} contributes. The power counting also justifies an iterative solution of the Schrödinger equation for $r > R$ along the lines of a conventional Born series. It differs from the usual Born series in that the boundary conditions at R are implemented. Finally, Q counting is used in expanding out the final expression for $k\cot(\delta)$.

Carrying out this program gives the following expression for $k\cot(\delta)$ at order Q^2 for the $^1S_0$ channel

$$p\cot(\delta) = -\frac{1}{a_0} + m_\pi^2 \left[d + \frac{g_A^2 M}{16\pi f_\pi^2}\,(\gamma + \ln(m_\pi R)) \right]$$

$$+ \frac{1}{2}\,r_e^0\,p^2 - \frac{g_A^2 M}{64\pi a_0^2 f_\pi^2}\,\left(\frac{m_\pi^2}{p^2}\right)\ln\left(1 + \frac{4p^2}{m_\pi^2}\right)$$

$$+ \frac{g_A^2 m_\pi M}{16\pi a_0 f_\pi^2}\,\left(\frac{m_\pi}{p}\right)\tan^{-1}\left(\frac{2p}{m_\pi}\right) + \frac{g_A^2 m_\pi^2 M}{64\pi f_\pi^2}\,\ln\left(1 + \frac{4p^2}{m_\pi^2}\right) \qquad (8)$$

The convention used here has $f_\pi = 93$ MeV. Apart from well-known parameters from pionic physics, there are three parameters—a_0, d and r_e^0, where $1/a_{\text{short}}$ from eq. (7) is rewritten as $1/a_0 + dm_\pi^2$ with $1/a_0 \sim Q$, and $dm_\pi^2 \sim Q^2$. These parameters fix the energy dependence of the wave function at the matching

scale R; renormalization group invariance requires d to depend on R logarithmically. This form is precisely equivalent to the calculation in PDS, provided that the following identifications are made between the coefficients used above and those used in PDS with the notation of ref. 1.

$$\frac{4\pi}{M} \frac{1}{-\mu + 1/a_0} = C_0$$

$$\frac{1}{2} r_e^0 = \frac{C_2 M}{4\pi} \left(\mu^2 - \frac{2\mu}{a_0} + \frac{1}{a_0^2} \right)$$

$$m_\pi^2 \left[d + \frac{g_A^2 M}{16\pi f_\pi^2} \left(\gamma + \ln(m_\pi R) \right) \right] =$$

$$\frac{g_A^2 M}{16\pi f_\pi^2} \left(m_\pi^2 \ln \left(\frac{m_\pi}{\mu} \right) - m_\pi^2 + \frac{1}{a_0^2} - 2\frac{\mu}{a_0} + \mu^2 \right)$$

$$+ \quad \frac{D_2 M}{4\pi} \left(m_\pi^2 \mu^2 - \frac{2m_\pi^2 \mu}{a_0} + \frac{m_\pi^2}{a_0^2} \right) \tag{9}$$

Formally, this is encouraging in the sense that it explicitly demonstrates the scheme independence of physical quantities. At the same time, there is an important hint of trouble which may lie ahead. In doing the derivation the matching scale, R was taken to scale as Q^0 and the quantity $m_\pi R$ as order Q^1. To obtain the final expression only the leading term in $m_\pi R$ is kept. If $R \sim 1/m_\pi$ this is clearly problematic, and from the previous discussion about potentials, we see that $m_\pi R \sim 1$.

It should also be stressed that the quantity $p \cot(\delta)$ is an extremely useful observable to work with in Q counting. Unlike the amplitude itself, there are no poles near $p = 0$; thus the issues of reorganizing the expansion as in OS [4] do not come up. Moreover, the expression is valid near $p = 0$ (assuming that Q counting holds) so it should be useful for ultra low energy scattering.

4 Low Energy Theorems

One difficulty with eq. (8) is that it is given in terms of a_0 which is the scattering length for the short distance potential only; as such it is not an observable. However, one can express everything in terms of physical observables and in doing so develop "low energy theorems" [8]. The trick is to rel ate a_0 to the

physical scattering length as follows:

$$-\frac{1}{a} = -\frac{1}{a_0} + m_\pi^2 \left[d + \frac{g_A^2 M}{16\pi f_\pi^2} \left(\gamma + \ln(m_\pi R)\right)\right] + \frac{g_A^2 M}{16\pi f_\pi^2} \left(\frac{2m_\pi}{a_0} - \frac{1}{a_0^2}\right)$$

$$= -\frac{1}{a_0} + \mathcal{O}(Q^2/\Lambda) \tag{10}$$

Therefore in all of the $\mathcal{O}(Q^2)$ terms in eq. (8) one can replace a_0 by the physical a; the error in doing this is $\mathcal{O}(Q^3)$ which is one order beyond the order at which I am working. One gets the following expression orginally derived in ref. 8.

$$p\cot(\delta) = -\frac{1}{a_0} + m_\pi^2 \left[d + \frac{g_A^2 M}{16\pi f_\pi^2} \left(\gamma + \ln(m_\pi R)\right)\right]$$

$$+ \frac{1}{2} r_e^0 p^2 - \frac{g_A^2 M}{64\pi a^2 f_\pi^2} \left(\frac{m_\pi^2}{p^2}\right) \ln\left(1 + \frac{4p^2}{m_\pi^2}\right)$$

$$+ \frac{g_A^2 m_\pi M}{16\pi a f_\pi^2} \left(\frac{m_\pi}{p}\right) \tan^{-1}\left(\frac{2p}{m_\pi}\right) + \frac{g_A^2 m_\pi^2 M}{64\pi f_\pi^2} \ln\left(1 + \frac{4p^2}{m_\pi^2}\right) \tag{11}$$

One can expand this as a Taylor series in p to obtain ERE coefficients. They are given by:

$$v_2 = \frac{g_A^2 M}{16\pi f_\pi^2} \left(-\frac{16}{3a^2 m_\pi^4} + \frac{32}{5a\, m_\pi^3} - \frac{2}{m_\pi^2}\right)$$

$$v_3 = \frac{g_A^2 M}{16\pi f_\pi^2} \left(\frac{16}{a^2 m_\pi^6} - \frac{128}{7a\, m_\pi^5} + \frac{16}{3m_\pi^4}\right)$$

$$v_4 = \frac{g_A^2 M}{16\pi f_\pi^2} \left(-\frac{256}{5a^2 m_\pi^8} + \frac{512}{9a\, m_\pi^7} - \frac{16}{m_\pi^6}\right)$$

$$\ldots \tag{12}$$

The preceding low energy theorems are valid to leading nontrivial order in Q counting; corrections are of *relative* order Q/Λ.

Several features of these expressions are notable. The first is that they are true predictions, independent of choices made in fitting, to this order in Q counting. As such they are low energy theorems. One consequence of this is that different schemes (*eg.* fitting out the pole as in OS rather than using $1/a$) will only give corrections at relative order Q/Λ. In this sense these predictions can be considered "low energy theorems " which become exact in the limit

Table 1: A comparison of the predicted effective range expansion coefficients, v_i, for the $^1S_0$ and $^3S_1$ channels with coefficients extracted from the Nijmegen partial wave analysis.

	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)
δ ($^1S_0$ channel)			
low energy theorem	-3.3	17.8	-108.
partial wave analysis	-.48	3.8	-17.
δ ($^3S_1$ channel)			
low energy theorem	-.95	4.6	-25.
partial wave analysis	.04	.67	-4.0

$(m_\pi, 1/a)/\Lambda \to 0$. The second significant fact is that all terms for all the expressions for the v_i coefficients diverge in the chiral limit of $m_\pi \to 0$. This implies that these quantities are dominated by pionic effects and hence are a good place to test whether the pionic parts of Q counting are working.

If the pionic parts of Q counting were under control one would expect that these predictions would work well. In practice, however, they work quite poorly. This can be seen in Table (1) where the prediction from the low energy theorems are compared with coefficients extracted from the Nijmegen partial wave analysis (PWA). The prediction from the low energy theorems are typically off by a factor of

5 or so. This suggests that pionic parts of the Q counting are failing rather badly.

One possible difficulty with the comparison of the v_i coefficients from the low energy theorems with the experimental data is that there is no experimental data. The coefficients extracted from the Nijmegen PWA were based on a fit to the smoothed "best fit". In principle one should do this fit including an error analysis based on the uncertainties. This cannot be done from the published data of the Nijmegen group as they did not publish information about correlated errors. Thus, one might wonder whether it is meaningful to extract high derivatives which are presumably rather sensitive to errors. A simple error estimate in ref. 4 concludes that the errors are likely to be too large to get any quantitative information about the v coefficients. From this one might conclude the disagreement between the low energy theorems and the "data" in Table (1) is due to an inability to extract the v from the scattering data in a reliable way. This is almost certainly not the case, however.

Table 2: A comparison of the effective range expansion coefficients, v_i, for the $^3S_1$ and $^3S_1$ channel predicted from the low energy theorem with coefficients extracted from the partial wave analysis and with three potential models—Nijmegen I, Nijmegen II and Reid 93—which were fit directly to the scattering data.

δ ($^3S_1$ channel)	v_2 (fm^3)	v_3 (fm^5)	v_4 (fm^7)
low energy theorem	-.95	4.6	-25.
partial wave analysis	.040	.672	-3.96
Nijm I	.046	.675	-3.97
Nijm II	.045	.673	-3.95
Reid93	0.033	.671	-3.90

The Nijmegen group made several independent fits to the scattering data. One was the PWA. The others were various potential models which were fit directly to the data (*i.e.* not to the PWA). These fits had a χ^2 per degree of freedom of essentially unity. Thus they can be regarded as independent fits to the data[10]. As the potential models have different forms from each other, they clearly have different systematic errors. Moreover in doing the least squares fit different models make different compromises in fitting individual data points so that they tend to explore the statistical errors. Thus, one might expect that the spread in the coefficients extracted in the different fits gives a reasonable feel for the scale of the uncertainty. Table (2) shows the v_i coefficients for these fits for the triplet channel[11], and it is manifestly clear that the spread in the effective range parameters as extracted from the three is vastly smaller than the difference with the predictions from the low energy theorems.

5   Re-summing the Effective Range Expansion

The effective range expansion parameters discussed in the previous section provide a dramatic way to see that the pionic parts of Q counting may have serious problems with convergence. However, there are a number of drawbacks with looking at the v_i coefficients. As noted above, there are ambiguities in the extraction from the data and it is hard to get reliable error bars. Moreover, the effective range expansion itself has a very limited radius of convergence. Because of the pion cut one expects the effective range expansion to converge only for $p < m_\pi/2$.

Of course, all of the low energy theorems for the v_i coefficients are contained in eq. (11). We can study this directly without expanding as a function

Table 3: A comparison of the shape function $\mathcal{S}(p^2) = p \cot(\delta) + 1/a - 1/2 r_e p^2$ for the ${}^3S_1$ channel extracted from the Nijmegen partial wave analysis with the prediction by the low energy theorem.

lab energy (MeV)	$\mathcal{S}$ extracted (Mev)	$\mathcal{S}$ low energy theorem (Mev)
Deuteron Pole	-0.0017 ± 0.0125	-0.743
1	-0.00095 ± 0.00721	-0.0258
5	0.0428 ± 0.0194	-0.535
10	0.245 ± 0.047	-1.78
25	2.18 ± 0.14	-7.54
50	11.03 ± 0.24	-20.10

of k. In effect, this amounts to re-summing the effective range expansion and using $p \cot(\delta)$ as our fundamental quantity. There are two obvious advantages to doing this: First, one can avoid the problem of extracting the v_i coefficients from noisy data and instead we can compare directly with the partial wave analysis (which includes error estimates). Second, one is no longer restricted to $p < m_\pi/2$ since the re-summed expression is valid over the entire domain of Q counting. Unfortunately, $p \cot(\delta)$ does not isolate the pionic contributions from the rest, and at low p it is dominated by $1/a$ physics and the fitting procedure which gives the effective range. There is a clean way to finesse this problem, however. Rather than study $p \cot(\delta)$ directly, one can study the following "shape function"[12]:

$$\mathcal{S}(p) = p \cot(\delta) - (-1/a + 1/2 r_e p^2) \tag{13}$$

which is just the re-summed effective range expansion with the first two terms subtracted off. This has the advantage of removing completely the sensitivity to $1/a$ and the fitting of the effective range. The quantity is completely pion dominated since in a theory with pions integrated out $\mathcal{S}(p)$ is $\mathcal{O}(Q^3)$, while in a theory with explicit pions it is $\mathcal{O}(Q^2)$.

In Table (3) we see the low energy theorem prediction for $\mathcal{S}(p)$ compared with values extracted from the Nijmegen partial wave analysis for the triplet channel. Note the disagreement is quite pronounced. Moreover, note that error estimates are given for the results extracted from the scattering data. It is manifestly clear that the discrepancies are *not* due to uncertainties in the data. Again this suggests that the pionic parts of Q counting are not predictive at leading nontrivial order—at least not for this observable.

6 Conclusions

The development of Q counting is an extremely important step in our understanding of effective field theory for nuclear physics. However, as stressed in this talk Q counting involves two small mass parameters, $1/a$ and m_π. While there is every indication that the part of the theory based on expanding in $1/(a\Lambda)$ is working well, the chiral counting is far more problematic. The expansion has no predictive power for the effective range parameters and the shape function at leading nontrivial order (NLO). As both of these quantities are chirally sensitive this failure suggests that the chiral expansion may not be well under control.

There are a number of possibilities. The most optimistic one is simply that the NLO calculation is not adequate, and if one works at higher order all will be well. The most pessimistic possibility is that that the chiral expansion is not converging and that this failure is general. Clearly, the way to resolve the situation is to work at higher order. It is important when doing so, however, to focus on observables which are highly sensitive to the pion physics.

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